

# How DC Brushed Motors Work (Another example of a first-order system)

## 1. Introduction to DC Brushed Motors

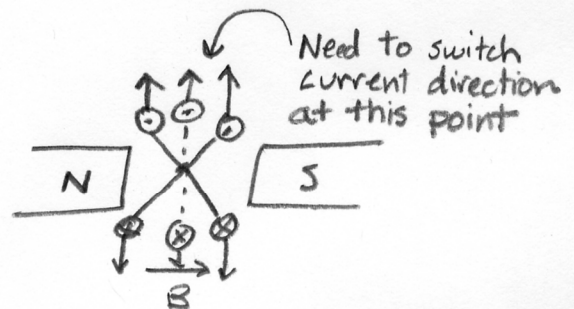
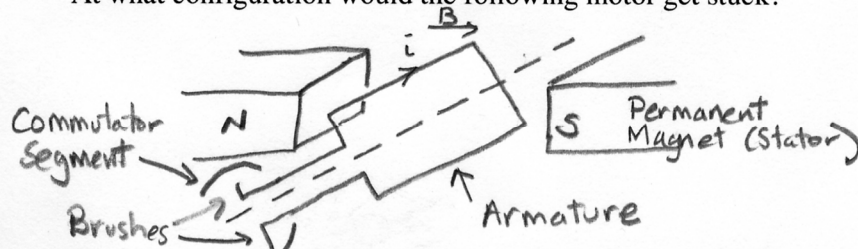
- very common for small jobs (toys, some appliances, robots)
- invented by Michael Faraday in the 1850's
- Operating principle:
  - apply voltage, motor spins
  - Polarity of voltage determines motor direction
  - Amplitude of voltage determines motor speed
- Other motor types: AC motors (washing machine), DC brushless motors, DC stepper motors

## 2. Physics of Operation

a. Makes use of Lorentz Force Law:  $\vec{F} = i\vec{l} \times \vec{B}$

where  $F$  = force,  $l$  = unit vector in direction of current flow,  $B$  = magnetic flux,  $i$  = current into motor  
i.e. current-carrying conductors placed in magnetic fields create forces

At what configuration would the following motor get stuck?



Use "commutation" to reverse current direction and keep motor turning

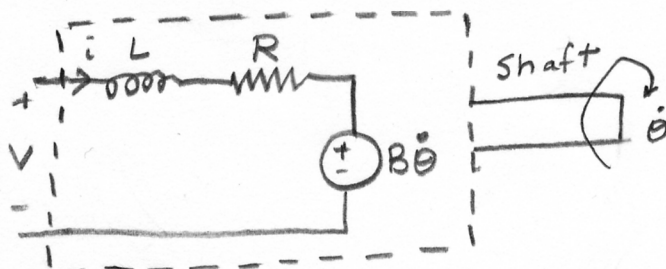
Adding enough commutator segments gives:  $\tau = Bi$ , where  $B$  = "torque constant"

b. Back EMF

- back EMF" (electromotive force or 'voltage')
- the voltage produced by motor as a result of its speed
- voltage is proportional to speed  $V = B\dot{\theta}$
- physical basis: armature windings are an inductor
- as motor spins, get  $di/dt$  in armature
- $V = L di/dt \propto$  angular velocity
- Can use a motor as a velocity sensor (i.e. a "tachometer") by measuring voltage across terminals
- This is also the principle used by generators.
- Real tachometers have many armature coils to reduce voltage ripple

## 3. Mathematical Model of a DC Brushed Motor

A motor has a resistance and inductance associated with its coils.



Electrical Side

$$V = L \frac{di}{dt} + Ri + B\dot{\theta}$$

Mechanical Side

$$\tau = Bi$$

To see how the model predicts the motor behavior, consider two cases:

Case 1) Hold shaft fixed, apply constant voltage. What is the motor torque as a function of time?

Assume  $i(0)=0$ ; shaft fixed  $\Rightarrow \dot{\theta}=0$

$$V = L \frac{di}{dt} + Ri + B\dot{\theta} \Rightarrow 0$$

Solve 1st order DEQ

1. General soln:  $\frac{di}{dt} = -\frac{R}{L}i \Rightarrow i = K e^{-t/\tau_c}$   $\tau_c = \frac{L}{R} = \text{time constant}$

2. Part soln: Assume  $i = \text{constant} = A$

$$L \frac{dA}{dt} + RA = V \Rightarrow A = \frac{V}{R}$$

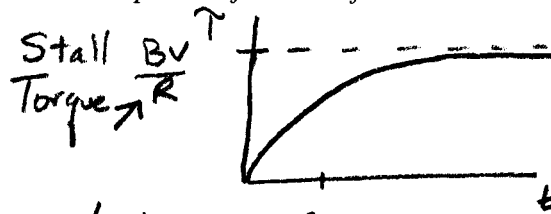
3. Total:  $i = \frac{V}{R} + K e^{-t/\tau_c}$   
Find  $K \Rightarrow i(0)=0 = \frac{V}{R} + K \Rightarrow K = -\frac{V}{R}$

Observations:

The stall torque = the torque you feel if you hold the motor shaft fixed

It takes time for a motor to develop torque (describable with a time constant)

After the transient response, the motor acts like a resistor



Soln:  
 $i = \frac{V}{R} (1 - e^{-t/\tau_c})$   
Torque  $\tau = Bi$   
 $\tau = \frac{BV}{R} (1 - e^{-t/\tau_c})$

Case 2) Allow shaft to spin freely, apply constant voltage. What is the motor speed as a function of time?

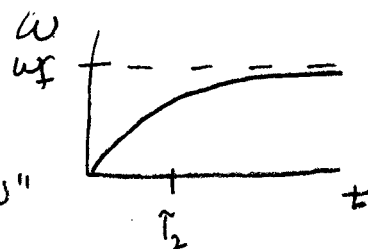
Assume shaft has inertia  $\tau = J\ddot{\theta}$ , assume  $\frac{di}{dt} \approx 0$  (current reaches steady state)

$$V = L \frac{di}{dt} + Ri + B\dot{\theta} \Rightarrow V = Ri + B\dot{\theta} \quad \text{recall } \tau = Bi \Rightarrow i = \frac{\tau}{B} = \frac{J\ddot{\theta}}{B}$$

$$\frac{RJ}{B} \ddot{\theta} + B\dot{\theta} = V \quad \text{let } \omega = \dot{\theta}$$

$$\frac{RJ}{B} \dot{\omega} + B\omega = V \quad \text{SOLUTION: } \omega = \frac{V}{B} (1 - e^{-t/\tau_2}) \quad \tau_2 = \frac{RJ}{B^2}$$

as  $t \rightarrow \infty$ ,  $\omega \rightarrow \omega_f = \frac{V}{B}$  "No Load Speed"



Observations:

No load speed is independent of inertia and proportional to voltage

Time constant of speed increase depends on inertia

Motor requires no power at no load speed (actually does because of friction)

at  $\omega = \dot{\theta} = \omega_f$

$$V = Ri + B\dot{\theta}$$

$$V = Ri + B \frac{V}{B} \Rightarrow i = 0$$

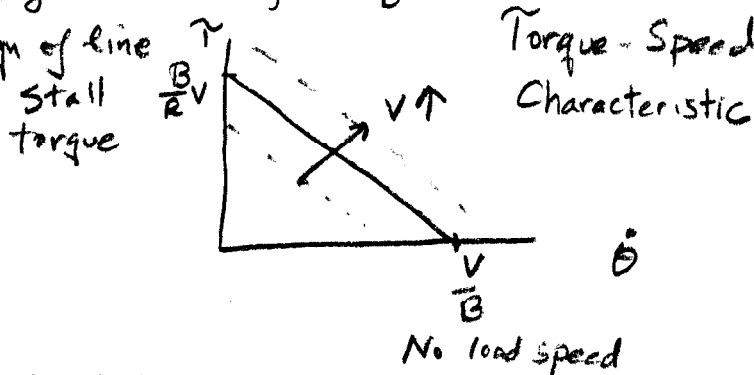
Summary: Torque-speed curve for a DC brushed motor

For  $V = \text{constant}$  ( $\frac{di}{dt} = 0$ ) you get  $V = Ri + B\dot{\theta}$ ,  $i = \frac{\tau}{B}$

$V = \frac{R}{B} \tau + B\dot{\theta} \rightarrow$  can be written in eqn of line

$$\tau = -\frac{B^2}{R} \dot{\theta} + \frac{B}{R} V$$

$$y = mx + b$$



Important Ideas:

- Lorentz force law
- Commutation
- Back EMF
- Mathematical model of motor (inductor, resistor, back EMF)
- Exponential increase in torque if shaft is fixed; in speed if shaft is free to spin
- Torque-speed curve (no-load speed, stall torque)